Benha University | **Electrical Circuits (2)** | **Electrical Eng. Dept.**
Faculty of Engineering | | **1st year communication**
Shoubra | | **15-17 March 2015**

Sheet (4)... Parallel Resonance (Solution)

1. Consider the circuit shown in Figure 1.

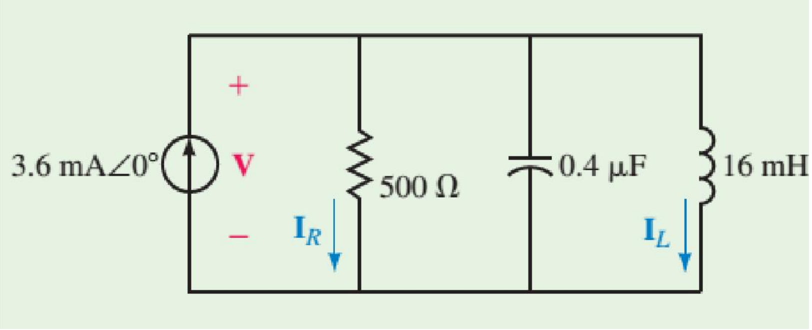

Fig. 1

a. Determine the resonant frequencies, ω_P(rad/s) and f_P(Hz) of the tank circuit.

b. Find the Q of the circuit at resonance.

c. Calculate the voltage across the circuit at resonance.

d. Solve for currents through the inductor and the resistor at resonance.

e. Determine the bandwidth of the circuit in both radians per second and hertz.

f. Sketch the voltage response of the circuit, showing the voltage at the half-power frequencies.

g. Sketch the selectivity curve of the circuit showing P(watts) versus ω(rad/s).

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a. $$\omega_P = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(16\text{ mH})\,(0.4\ \mu\text{F})}} = 12.5\text{ krad/s}$$

$$f_P = \frac{\omega}{2\pi} = \frac{12.5\text{ krad/s}}{2\pi} = 1989\text{ Hz}$$

b. $$Q_P = \frac{R_P}{\omega L} = \frac{500\ \Omega}{(12.5\text{ krad/s})\,(16\text{ mH})} = \frac{500\ \Omega}{200\ \Omega} = 2.5$$

c. At resonance, $\mathbf{V}_C = \mathbf{V}_L = \mathbf{V}_R$, and so

$$\mathbf{V} = \mathbf{IR} = (3.6\text{ mA}\angle 0°)\,(500\ \Omega\angle 0°) = 1.8\text{ V}\ \angle 0°$$

d. $$\mathbf{I}_L = \frac{\mathbf{V}_L}{\mathbf{Z}_L} = \frac{1.8\text{ V}\angle 0°}{200\ \Omega\angle 90°} = 9.0\text{ mA}\angle -90°$$

$$\mathbf{I}_R = \mathbf{I} = 3.6\text{ mA}\angle 0°$$

e. $$\text{BW(rad/s)} = \frac{\omega_P}{Q_P} = \frac{12.5\text{ krad/s}}{2.5} = 5\text{ krad/s}$$

$$\text{BW(Hz)} = \frac{\text{BW(rad/s)}}{2\pi} = \frac{5\text{ krad/s}}{2\pi} = 795.8\text{ Hz}$$

f. The half-power frequencies are calculated from Equations 21–48 and 21–49 since the Q of the circuit is less than 10.

$$\omega_1 = -\frac{1}{2RC} + \sqrt{\frac{1}{4R^2C^2} + \frac{1}{LC}}$$
$$= -\frac{1}{0.0004} + \sqrt{\frac{1}{1.6\times 10^{-7}} + \frac{1}{6.4\times 10^{-9}}}$$
$$= -2500 + 12\,748$$
$$= 10\,248\text{ rad/s}$$

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$$\omega_2 = \frac{1}{2RC} + \sqrt{\frac{1}{4R^2C^2} + \frac{1}{LC}}$$
$$= \frac{1}{0.0004} + \sqrt{\frac{1}{1.6 \times 10^{-7}} + \frac{1}{6.4 \times 10^{-9}}}$$
$$= 2500 + 12\,748$$
$$= 15\,248 \text{ rad/s}$$

The resulting voltage response curve is illustrated in Figure 21–32.

g. The power dissipated by the circuit at resonance is

$$P = \frac{V^2}{R} = \frac{(1.8\text{ V})^2}{500\ \Omega} = 6.48 \text{ mW}$$

The selectivity curve is now easily sketched as shown in Figure 21–33.

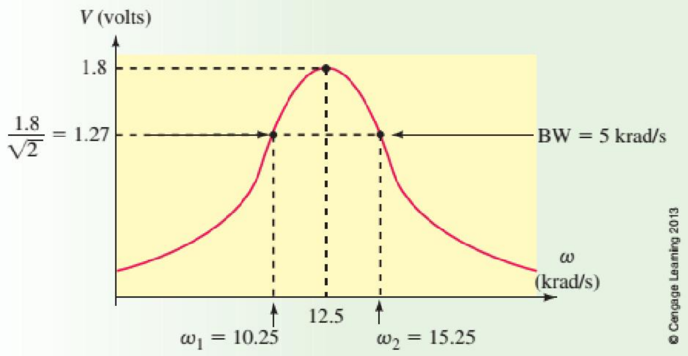

FIGURE 21–32

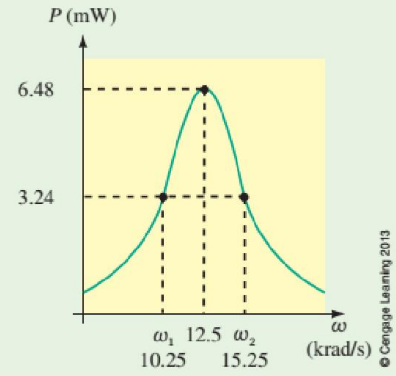

FIGURE 21–33

2. Consider the circuit of Figure 2.

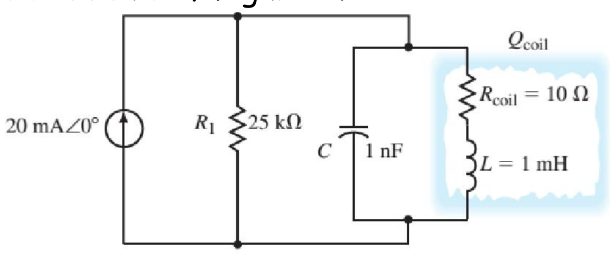

Fig. 2

a. Calculate the resonant frequency, ω_P, of the tank circuit.
b. Find the Q of the coil at resonance.
c. Sketch the equivalent parallel circuit.
d. Determine the Q of the entire circuit at resonance.
e. Solve for the voltage across the capacitor at resonance.

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a. Since the ratio $L/C = 1000 \geq 100R_{coil}$, we use the approximation:

$$\omega_P = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1\text{ mH})(1\text{ nF})}} = 1\text{ Mrad/s}$$

b. $$Q_{coil} = \frac{\omega L}{R_{coil}} = \frac{(1\text{ Mrad/s})(1\text{ mH})}{10\,\Omega} = 100$$

c. $$R_P \cong Q_{coil}^2 R_{coil} = (100)^2(10\,\Omega) = 100\text{ k}\Omega$$

$$X_{LP} \cong X_L = \omega L = (1\text{ Mrad/s})(1\text{ mH}) = 1\text{ k}\Omega$$

The circuit of Figure 21–35 shows the circuit with the parallel equivalent of the inductor.

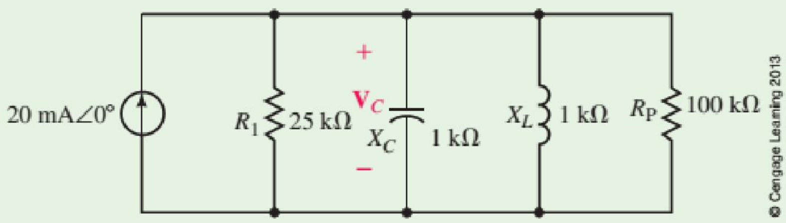

FIGURE 21–35

We see that the previous circuit may be further simplified by combining the parallel resistances:

$$R_{eq} = R_1 \| R_P = \frac{(25\text{ k}\Omega)(100\text{ k}\Omega)}{25\text{ k}\Omega + 100\text{ k}\Omega} = 20\text{ k}\Omega$$

The simplified equivalent circuit is shown in Figure 21–36.

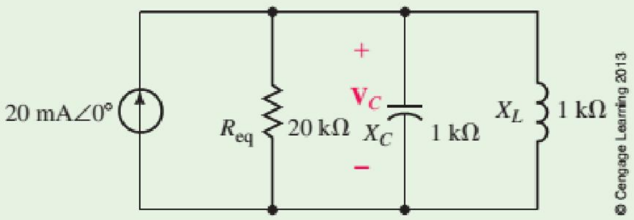

FIGURE 21–36

d. $$Q_P = \frac{R_{eq}}{X_L} = \frac{20\text{ k}\Omega}{1\text{ k}\Omega} = 20$$

e. At resonance,

$$\mathbf{V}_C = \mathbf{I}R_{eq} = (20\text{ mA}\angle 0°)(20\text{ k}\Omega) = 400\text{ V}\angle 0°$$

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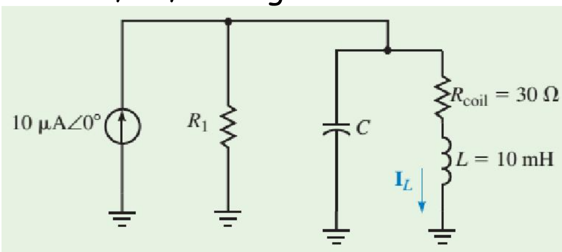
3. Determine the values of R1and C for the resonant tank circuit of Figure 3 so that the given conditions are met.
   L=10 mH, Rcoil=30Ω, fP=58 kHz, BW =1 kHz
   Solve for the current, IL, through the inductor.



$$Q_P = \frac{f_P}{BW(Hz)} = \frac{58\text{ kHz}}{1\text{ kHz}} = 58$$

$$\omega_P = 2\pi f_P = (2\pi)(58\text{ kHz}) = 364.4\text{ krad/s}$$

$$C = \frac{1}{\omega_P^2 L} = \frac{1}{(364.4\text{ krad/s})^2(10\text{ mH})} = 753\text{ pF}$$

$$Q_{coil} = \frac{\omega_P L}{R_{coil}}$$

$$= \frac{(364.4\text{ krad/s})(10\text{ mH})}{30\ \Omega}$$

$$= \frac{3.644\text{ k}\Omega}{30\ \Omega} = 121.5$$

$$R_P \cong Q_{coil}^2 R_{coil} = (121.5)^2(30\ \Omega) = 443\text{ k}\Omega$$

$$X_{LP} \cong X_L = 3644\ \Omega$$

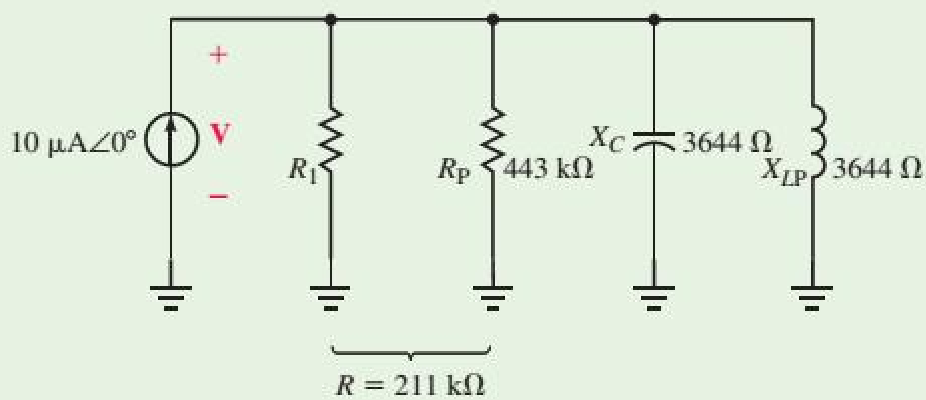


Fig . 3

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The quality factor Q_P is used to determine the total resistance of the ci

$$R = Q_P X_C = (58)(3.644\ \text{k}\Omega) = 211\ \text{k}\Omega$$

But

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_P}$$

$$\frac{1}{R_1} = \frac{1}{R} - \frac{1}{R_P} = \frac{1}{211\ \text{k}\Omega} - \frac{1}{443\ \text{k}\Omega} = 2.47\ \mu\text{S}$$

And so

$$R_1 = 405\ \text{k}\Omega$$

The voltage across the circuit is determined to be

$$\mathbf{V} = \mathbf{I}R = (10\ \mu\text{A}\angle 0^\circ)(211\ \text{k}\Omega) = 2.11\ \text{V}\angle 0^\circ$$

and the current through the inductor is

$$\mathbf{I}_L = \frac{\mathbf{V}}{R_{\text{coil}} + jX_L} = \frac{2.11\ \text{V}\angle 0^\circ}{30 + j3644\ \Omega} = \frac{2.11\ \text{V}\angle 0^\circ}{3644\ \Omega\angle 89.95^\circ} = 579\ \mu\text{A}\angle -89.9$$

4. Let V_s= 20 cos(at) V in the circuit of Fig. 4. Find ω_o, Q, and B, as seen by the capacitor.

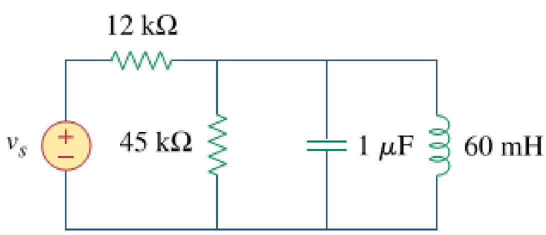
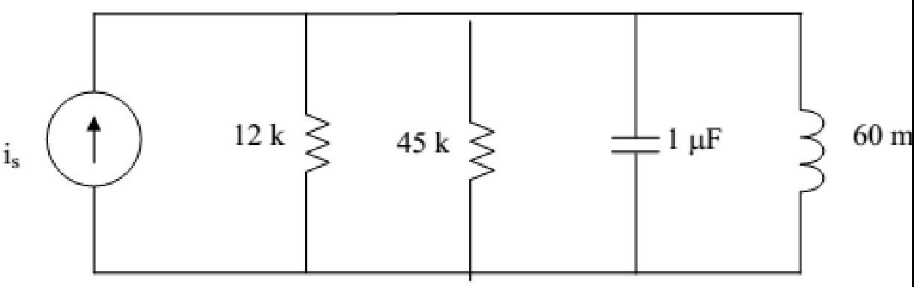


Fig. 4

We convert the voltage source to a current source as shown below.



$i_s = \frac{20}{12}\cos\omega t$, R = 12//45= 12x45/57 = 9.4737 kΩ

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{60x10^{-3}x1x10^{-6}}} = \underline{4.082\ \text{krad/s}}$$

$$B = \frac{1}{RC} = \frac{1}{9.4737x10^3x10^{-6}} = \underline{105.55\ \text{rad/s}}$$

$$Q = \frac{\omega_o}{B} = \frac{4082}{105.55} = \underline{38.674} = \underline{\mathbf{38.67}}$$

5. Design a parallel resonant RLC circuit with ωo= 10rad/s and Q = 20. Calculate the bandwidth of the circuit. Let R= 10Ω.

Select R = 10 Ω.

$$L = \frac{R}{\omega_0 Q} = \frac{10}{(10)(20)} = 0.05\ H = 50\ mH$$

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{(100)(0.05)} = 0.2\ F$$

$$B = \frac{1}{RC} = \frac{1}{(10)(0.2)} = 0.5\ rad/s$$

Therefore, if R = 10 Ω then

L = **50 mH**, C = **0.2 F**, B = **0.5 rad/s**

6. It is expected that a parallel RLC resonant circuit has a mid-band admittance of 25 ×10^{-3}S, quality factor of 80, and a resonant

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frequency of 200 krad/s. Calculate the values of R, L, and C. Find the bandwidth and the half-power frequencies.

At resonance,

$$Y = \frac{1}{R} \longrightarrow R = \frac{1}{Y} = \frac{1}{25\times10^{-3}} = \underline{\mathbf{40\ \Omega}}$$

$$Q = \omega_0 RC \longrightarrow C = \frac{Q}{\omega_0 R} = \frac{80}{(200\times10^{3})(40)} = \underline{\mathbf{10\ \mu F}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \longrightarrow L = \frac{1}{\omega_0^2 C} = \frac{1}{(4\times10^{10})(10\times10^{-6})} = \underline{\mathbf{2.5\ \mu H}}$$

$$B = \frac{\omega_0}{Q} = \frac{200\times10^{3}}{80} = \underline{\mathbf{2.5\ krad/s}}$$

$$\omega_1 = \omega_0 - \frac{B}{2} = 200 - 1.25 = \underline{\mathbf{198.75\ krad/s}}$$

$$\omega_2 = \omega_0 + \frac{B}{2} = 200 + 1.25 = \underline{\mathbf{201.25\ krad/s}}$$

7. For the "tank" circuit in Fig. 5, find the resonant frequency.

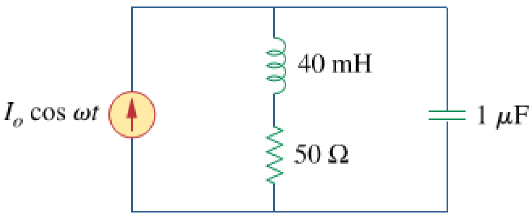

Fig. 5

$$Y = \frac{1}{R + j\omega L} + j\omega C = j\omega C + \frac{R - j\omega L}{R^2 + \omega^2 L^2}$$

At resonance, $\text{Im}(\mathbf{Y}) = 0$, i.e.

$$\omega_0 C - \frac{\omega_0 L}{R^2 + \omega_0^2 L^2} = 0$$

$$R^2 + \omega_0^2 L^2 = \frac{L}{C}$$

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = \sqrt{\frac{1}{(40\times 10^{-3})(1\times 10^{-6})} - \left(\frac{50}{40\times 10^{-3}}\right)^2}$$

$$\omega_0 = \underline{\mathbf{4841\ rad/s}}$$

Good Luck

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